Biostatistics

Factorial Design

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Introduction:

A factor is a variable that is controlled and varied during the course of an experiment. In a chemistry experiment, temperature and pressure may be the factors that are deliberately changed over the course of the experiment. In the clinical trial treatment can be a factor. A study of experimental therapy vs. placebo can be thought of as having a treatment factor with 2 levels, 0 or the study dosage. A study with two different treatments has the possibility of a two-way design, varying the levels of treatment A and treatment B.

Factorial clinical trials are experiments that test the effect of more than one treatment using a type of design that permits an assessment of potential interactions among the treatments.

In a factorial design there are two or more factors with multiple levels that are crossed, e.g., three dose levels of drug A and two levels of drug B can be crossed to yield a total of six treatment combinations:

low dose of A with low dose of B
low dose of A with high dose of B
mid dose of A with low dose of B
mid dose of A with high dose of B
high dose of A with low dose of B
high dose of A with high dose of B

Factorial designs offer certain advantages over conventional designs. There are a number of ways that you could look at these groups.

Characteristics of Factorial Designs

The simplest factorial design is the 2 × 2 factorial with two levels of factor A crossed with two levels of factor B to yield four treatment combinations. A special case of the 2 × 2 factorial with a placebo and an active formulation of factor A crossed with a placebo and an active formulation of factor B. This yields the four treatment regimens:
Placebo A + Placebo B
Placebo A + Active B
Active A + Placebo B
Active A + Active B
For example, here you could have a placebo for each treatment. In one case you might have a placebo injection for A and a placebo pill for B. Such a design allows the comparison of the levels of factor A (A main effects), the comparison of the levels of factor B (B main effects), and the investigation of A × B interactions.

What is the Factorial ANOVA?

ANOVA is short for **AN**alysis **O**f **VA**riance. As discussed in the chapter on the one-way ANOVA the main purpose of a one-way ANOVA is to test if two or more groups differ from each other significantly in one or more characteristics. A factorial ANOVA compares means across two or more independent variables. Again, a one-way ANOVA has one independent variable that splits the sample into two or more groups, whereas the factorial ANOVA has two or more independent variables that split the sample in four or more groups. The simplest case of a factorial ANOVA uses two binary variables as independent variables, thus creating four groups within the sample.

		Independent Variables	Independent Variables
		1	2+
Dependent Variables	1	One-way ANOVA	**Factorial ANOVA**
	2+	Multiple ANOVAs	MANOVA

These are both two-factor ANOVAs

This is an example of a **2 x 2** ANOVA

	B_1 Drug A	B_2 Drug B
A_1 Males	Sample 1	Sample 2
A_2 Females	Sample 3	Sample 4

This is an example of a **2 x 3** ANOVA

	B_1 80-degree room	B_2 90-degree room	B_3 100-degree room
A_1 Low Humidity	Sample 1	Sample 2	Sample 3
A_2 High Humidity	Sample 4	Sample 5	Sample 6

Stating the Hypotheses

Main Effect of A

$$H_0 : \mu_{A_1} = \mu_{A_2}$$
$$H_1 : \mu_{A_1} \neq \mu_{A_2}$$

Factor A has 2 levels
Factor B has 3 levels

Main Effect of B

$$H_0 : \mu_{B_1} = \mu_{B_2} = \mu_{B_3}$$
$$H_1 : \text{at least one mean is different from another}$$

Interaction Between A&B

$$H_0 : \text{There is no interaction between A \& B}$$
$$H_1 : \text{There is an interaction between A \& B}$$

Example 14.1

Data for a two-factor research study comparing two levels of task difficulty (easy and hard) and three levels of arousal (low, medium, and high).

The study involves a total of six different treatment conditions with $n = 5$ participants in each condition

		Factor *B* Arousal Level: Low	Factor *B* Arousal Level: Medium	Factor *B* Arousal Level: High	
Factor *A* Task Difficulty	Easy	3 1 1 6 4 $M = 3$ $T = 15$ $SS = 18$	1 4 8 6 6 $M = 5$ $T = 25$ $SS = 28$	10 10 14 7 9 $M = 10$ $T = 50$ $SS = 26$	$T_{ROW1} = 90$
	Difficult	0 2 0 0 3 $M = 1$ $T = 5$ $SS = 8$	2 2 2 2 $M = 3$ $T = 15$ $SS = 20$	1 7q 1 6 1 $M = 2$ $T = 10$ $SS = 20$	$T_{ROW2} = 30$
		$T_{COL1} = 20$	$T_{COL2} = 40$	$T_{COL3} = 60$	

$N = 30$
$G = 120$
$\Sigma X^2 = 860$

Summarizing Results

Source	*SS*	*df*	*MS*	*F*
Between treatments	260	5		
Factor A (difficulty)	120	1	120	F(1,24) = 24.00
Factor B (arousal)	80	2	40	F(2,24) = 8.00
A x B	60	2	30	F(2,24) = 6.00
Within treatments	120	24	5	
Total	380	29		